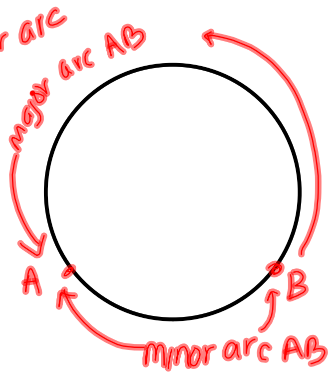
Section 8.3 - Properties of angles in a circle

Arc = is a section of the circumference of the circle.

→ shorter arc → minor arc

→ longer arc → major arc

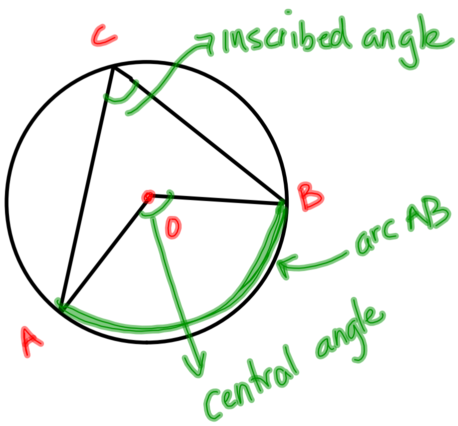


Central angle = is the angle formed by joining the endpoints of an arc to the center of the circle

ex: ∠ AOB is a central angle.

Inscribed angle = is the angle formed by joining the endpoints of an arc to a point on the circle

ex: ∠ ACB is an inscribed angle

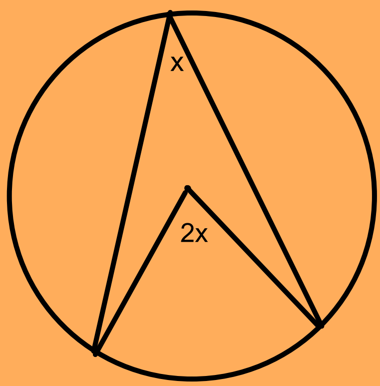


* the inscribed and central angles in this circle are subtended by the minor arc AB.

Section 8.3 Properties of Angles in a Circle

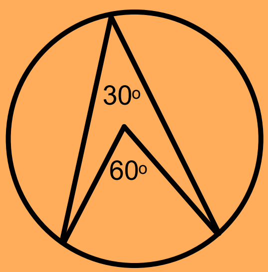
Central Angle and Inscribed Angle Property

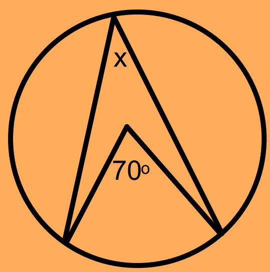
The measure of a central angle is twice the measure of an inscribed angle subtended by the same arc.

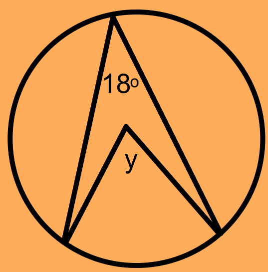


Examples

#1 #2 #3

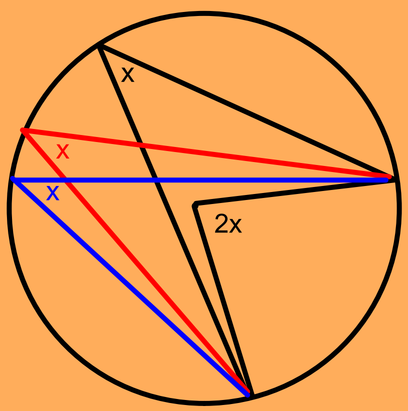






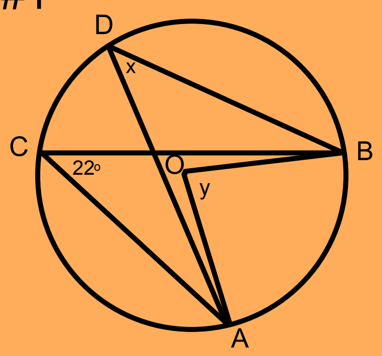
Inscribed Angles Property

Inscribed angles subtended by the same arc are equal.



Examples

#1



$\angle ACB$ and $\angle ADB$ are inscribed angles subtended by the same arc AB.
So, $\angle ACB = \angle ADB$
x = 22o

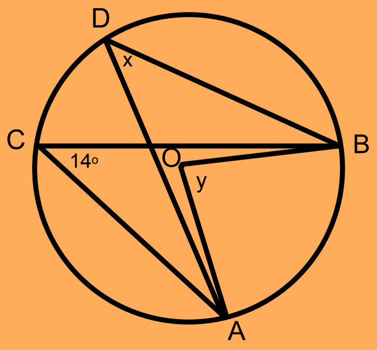
Central $\angle AOB$ and inscribed $\angle ACB$ are both subtended by arc AB

$\angle AOB = 2\angle ACB$

$y = 2 \times 22^{\circ}$

$y = 44^{\circ}$

#2



$\angle ACB$ and $\angle ADB$ are inscribed angles subtended by the same arc AB.
So, $\angle ACB = \angle ADB$
x = 14°

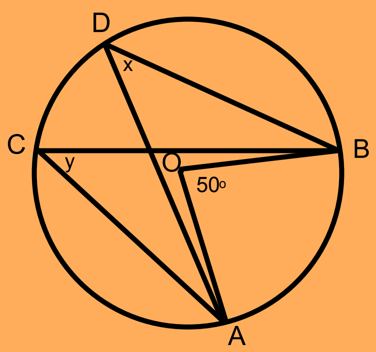
Central $\angle AOB$ and inscribed $\angle ACB$ are both subtended by arc AB

$\angle AOB = 2\angle ACB$

$y = 2 \times 14^{\circ}$

$y = 28^{\circ}$

#3



Since both inscribed angles are subtended from the same arc as the central angle

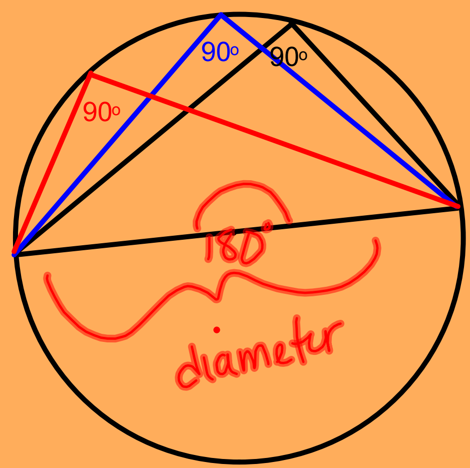
$$\angle ACB = \angle ADB = \frac{1}{2}\angle AOB$$

$$\therefore y = x = \frac{1}{2}(50^\circ)$$

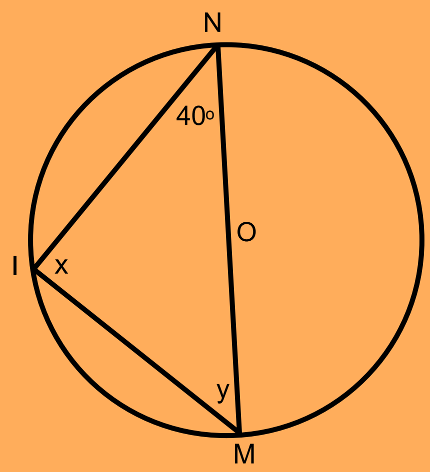
$$\therefore y = x = 25^\circ$$

Angles in a Semicircle Property

Inscribed angles subtended by a semicircle are right angles



Examples # 1

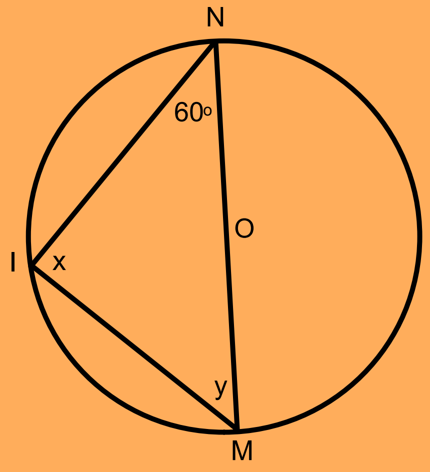


$\angle MIN$ is an inscribed angle subtended by a semicircle.
so, x = 90°

Since 3 angles in a triangle add to 180°,

y + 90 + 40 = 180
y + 130 = 180
y + 130 - 130 = 180 - 130
y = 50°

Example # 2 - Try this one!



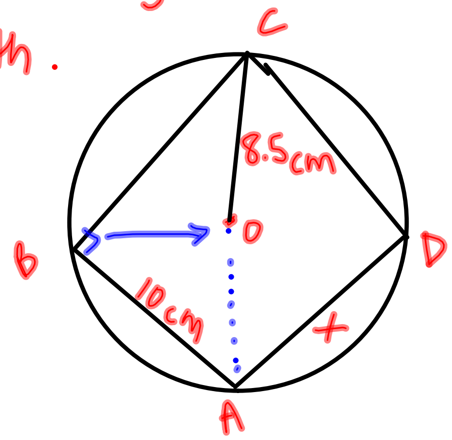
$\angle MIN$ is an inscribed angle subtended by a semicircle.
so, x = 90°

Since 3 angles in a triangle add to 180°,

y + 90 + 60 = 180
y + 150 = 180
y + 150 - 150 = 180 - 150
y = 30°

Example 3

Rectangle ABCD has its vertices on a circle with radius 8.5cm. The width of the rectangle is 10 cm. What is its length? Give the answer to the nearest tenth.



Solution: length of AD = x

$AC = 2 \times 8.5 = 17\text{cm}$ (diameter)

each ∠ in rectangle is 90°

we use the pythagorean theorem to get AD.

$a^2 + b^2 = c^2$

$x^2 + 10^2 = 17^2$

$x^2 + 100 = 289$

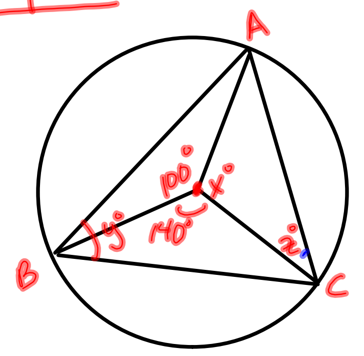
$x^2 = 289 - 100$

$x^2 = 189$

$x = \sqrt{189}$

$x = 13.7\text{ cm}$

Example 4:



Triangle ABC is inscribed in a circle, center O

$\angle AOB = 100°$ and $\angle COB = 140°$

Determine the values of $x°$, $y°$ and $z°$.

Solution:

$$100 + 140 + x = 360$$
$$240 + x = 360$$
$$x = 360 - 240$$
$$x = 120°$$

$\angle AOC$ is a central $\angle$

$\angle ABC$ is inscribed

$$y = \frac{1}{2}(120)$$
$$= 60°$$

OC and OA are radii, therefore $\triangle AOC$ is isosceles.

$$120 + z + z = 180°$$
$$2z = 180 - 120$$
$$\frac{2z}{2} = \frac{60}{2}$$
$$z = 30°$$

Practice Pages 410 - 412
#'s 3, 4, 5, 6, 11