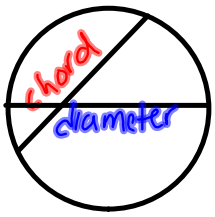
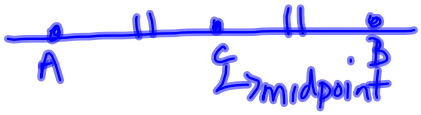
Section 8.2 – Properties of chords in a circle

chord = is a line segment that joins two points on a circle

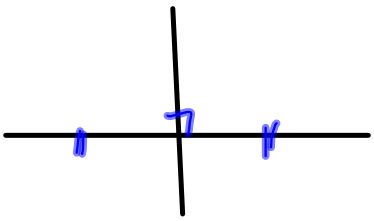


midpoint = is a point which divides a line segment into 2 equal parts.



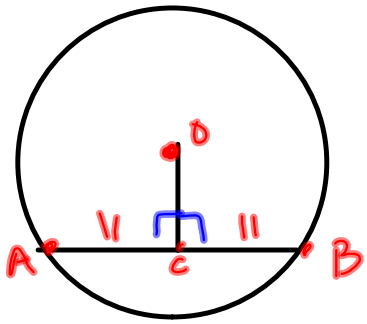
perpendicular = refer to line or line segments that meet at right angles (90°). Symbol is ⊥

perpendicular bisector = bisects a line segment at 90° and divides the line segment into 2 equal parts.



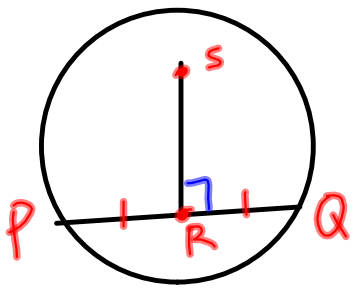
Perpendicular to chord Properties:

① the ⊥ from the center of a circle to a chord that is, the ⊥ dindes the chord into 2 equal parts.



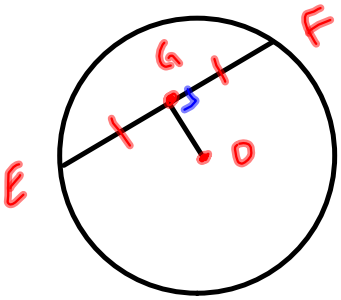
Point O is the center of the circle.
when $\angle OCB = \angle OCA = 90^\circ$
then $AC = CB$

② the ⊥ bisector of a chord in a circle passes through the center of the circle.



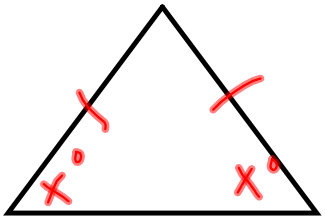
when $\angle SRQ = \angle SRP = 90^\circ$
and $PR = RQ$
then SR passes through O, the center of the circle.

③ A line that joins the center of a circle and the midpoint of a chord is ⊥ to the chord.



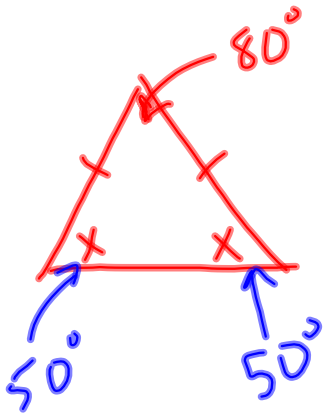
when O is the center of a circle and EG = GF, then $\angle OGE = \angle OGF = 90^\circ$.

*NOTE:



isosceles △

2 equal sides, 2 equal base angles.

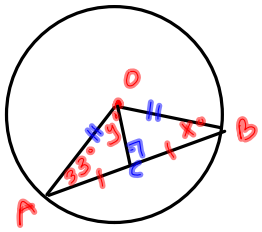


$180 - 80 = 100$

$\frac{100}{2} = 50^\circ$

A. Determining the measure of angles in a Δ

Example 5: Point O is the center of a circle, and line segment OC bisects chord AB. $\angle OAC = 33°$. Determine the values of $x°$ and $y°$.



Solution: Since OC bisects AB & passes through the center O. $OC \perp AB$.
therefore, $\angle OCA = \angle OCB = 90°$

$OA = OB$ radii we have an isosceles Δ.

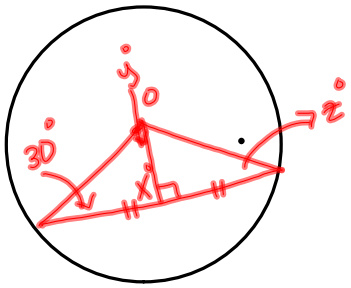
$x = 33°$

$y = ?$ Sum of ∠'s in Δ = 180°

$180 - 33 - 90 = 57°$

$y = 57°$

Example 6:



Solution:

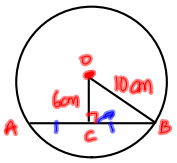
$x = 90°$

$y = 180° - 90° - 30° = 60°$

$z = 30°$

B) Using the pythagorean theorem

Example 1: O is the center of the circle
Find the length of chord AB.



Solution: $a^2+b^2=c^2$

$6^2+b^2=10^2$

$36+b^2=100$

$b^2=100-36$

$b^2=64$

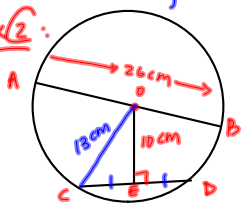
$b=\sqrt{64}$

$b=8$

So, BC = 8 cm

So the length of AB = 8x2 = 16 cm.

Ex 2:



Point O is the center of a circle.
AB is a diameter with length 26 cm.
CD is a chord that is 10 cm from the center of the circle.
What is the length of chord CD?
Give the answer to the nearest tenth.

Solution:

OC = 13 cm radius ($\frac{1}{2}$ diameter)

$OE \perp CD$ therefore OE bisects CD
which means CE = ED
use the pythagorean theorem

$a^2+b^2=c^2$

$a^2+10^2=13^2$

$a^2+100=169$

$a^2=169-100$

$a^2=69$

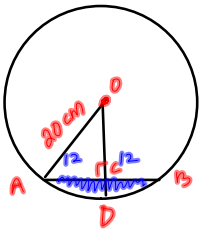
$a=\sqrt{69}$

$a=8.3$ cm

CE = 8.3 cm therefore CD = 2x8.3
= 16.6 cm

C. Solving problems using the property of a chord and its perpendicular.

Example: A horizontal pipe has a circular cross section, with center O. Its radius is 20 cm. Water fills less than one-half of the pipe. The surface of the water AB is 24 cm wide. Determine the maximum depth of the water, which is the depth CD.



Solution: Depth CD = OD − OC

OD = 20 cm radius

$OC \perp AB$ therefore, $AC = \frac{1}{2} AB$

AB = 24 cm, so AC = 12 cm

To get OC use the pythagorean theorem.

$a^2 + b^2 = c^2$

$a^2 + 12^2 = 20^2$

$a^2 + 144 = 400$

$a^2 = 400 - 144$

$a^2 = 256$

$a = \sqrt{256}$

$a = 16 \text{ cm} = OC$

$CD = OD - OC$

$= 20 - 16$

$= 4 \text{ cm}$

The maximum depth of the water is 4 cm.

Complete: pg 397 #'s 3-7, 10